Review of Mechanical Doll

Problem

The problem is to create a circuit of devices.

Devices have one or two exits each. In a circuit, devices are connected: each exit is connected to another (or possibly the same) device. The connections are decided by the contestants.

A ball is placed on one of the devices and travels through the circuit indefinitely. At each step, the ball leaves the present device through one of its exits and enters the connected device.

There are three types of devices: the origin, triggers, and switches.

The origin is where the ball starts.

Triggers are numbered from 1 to M. Contestants are given an array A of length N. The ball must visit triggers $A_0, A_1, \ldots, A_{N-1}, A_0, A_1, \ldots, A_{N-1}, A_0, A_1, \ldots$ in this order.

Switches have two exits each. The two exits work alternately. Contestants can decide the number of switches.

The task is to decide the connections so that the circuit satisfies the following conditions.

- The ball returns to the origin after a finite number of steps.
- By the time the ball first returns to the origin, every switch has been visited even times.
- The ball first returns to the origin after entering triggers exactly N times. The consecutive serial numbers of these triggers are $A_0, A_1, \ldots, A_{N-1}$.

The score is determined according to the number of switches. Contestants get the full score when $N + \log_2 N$ switches or less are used.

Subtasks and Solutions

Subtask 1 (2 points)

Each trigger appears in A at most once. $N + \log_2 N$ switches can be used.

- Connect triggers without switches.

Subtask 2 (4 points)

Each trigger appears in A at most twice. $N + \log_2 N$ switches can be used.

- Put a switch after each trigger which appears twice.

Subtask 3 (10 points)

Each trigger appears in A at most 4 times. $N + \log_2 N$ switches can be used.

- For each trigger which appears more than twice, use 3 switches to branch its exit into 4 exits (Imagine a balanced binary tree). For each trigger which appears 3 times, kill one out of the first 3 exits by connecting it to the "root" switch.

Subtask 4 (10 points)

$N = 16$. 20 switches can be used.

- Gather exits of all the triggers into one, and branch it into 16 exits with 15 (= 2^4 - 1) switches.

Subtask 5 (18 points)

There is only one trigger.

- (Solution A) ($2N$ switches, half score) Use $2^k - 1$ switches for k such that $2N > 2^k \geq N$, and kill excessive exits by connecting them to the "root" switch.
- ($N + \log_2 N$ switches, full score) In addition to Solution A, skillfully choose which exits to kill so that they are closely placed in the circuit. Then many switches are now unnecessary because their exits are both connected to the "root" switch.

Subtask 6 (56 points)

No additional constraints.

- ($2N$ switches, half score) Do Solution A for every trigger.
- (Solution B) ($2N$ switches, half score) Gather exits of all the triggers into one, and branch it using Solution A.
- ($N + \log_2 N$ switches, full score) In addition to Solution B, save switches by skillfully choosing which exits to kill.